Non-minimally coupled inflation via a conformal factor with a zero

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in collaboration with

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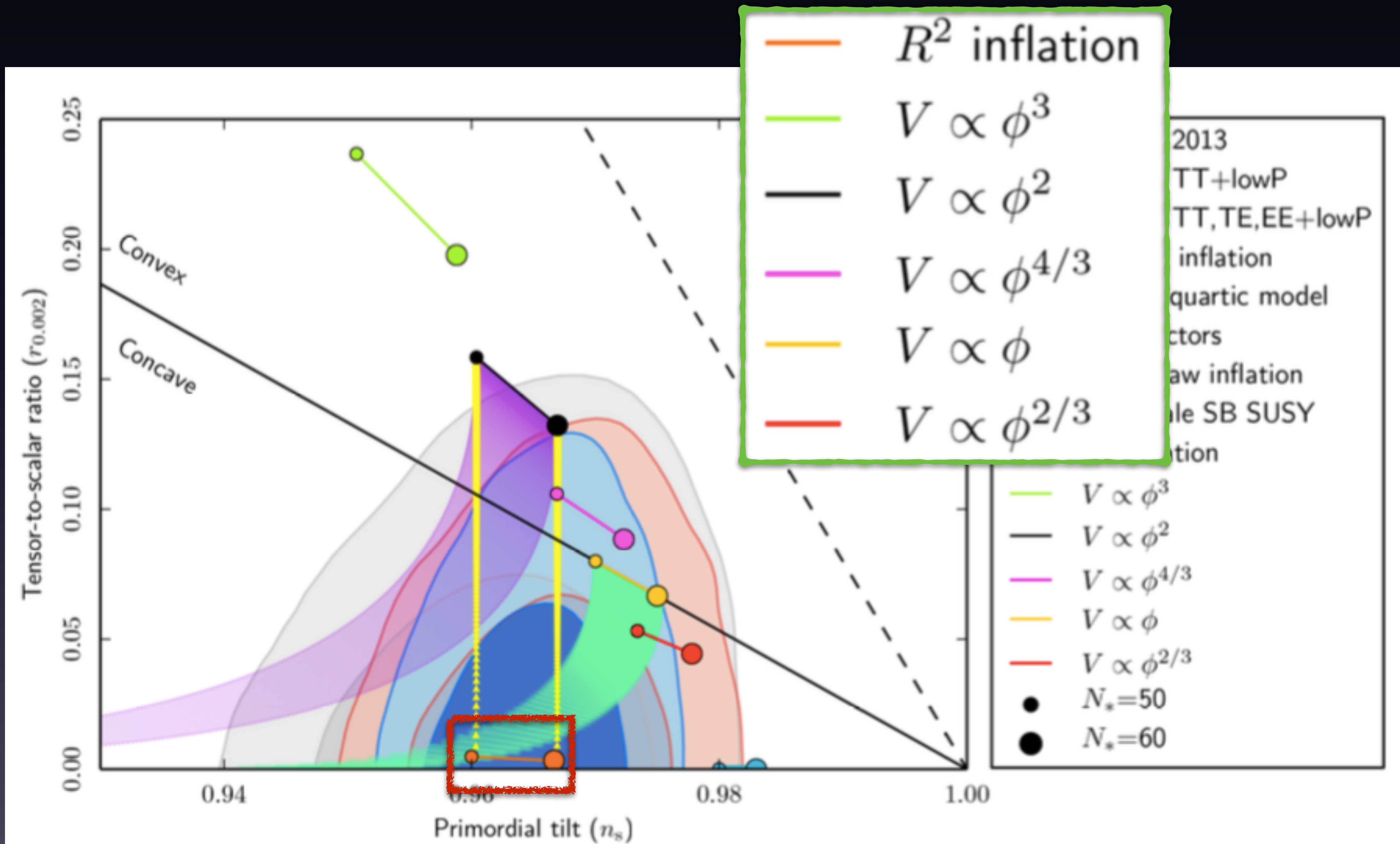
Non-minimal Coupling Model



Non-minimal Coupling Model

$$S_{\rm J} = \int d^4x \sqrt{-g_{\rm J}} \left[\frac{M_{\rm P}^2}{2} \Omega^2(\phi) R_{\rm J} - \frac{1}{2} g_{\rm J}^{\mu\nu} Z(\phi) \partial_\mu \phi \partial_\nu \phi - V_{\rm J}(\phi) \right]$$

Weyl rescaling (a.k.a. conformal transformation)

$$g_{\rm J}^{\mu\nu} \to g_{\rm E}^{\mu\nu} = \Omega^{-2} g_{\rm J}^{\mu\nu}$$

$$S_{\rm E} = \int d^4x \sqrt{-g_{\rm E}} \left[\frac{M_{\rm P}^2}{2} R_{\rm E} - \frac{1}{2} g_{\rm E}^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V_{\rm E}(\varphi) \right]$$

$$\left(\frac{d\varphi}{d\phi}\right)^2 = \frac{Z}{\Omega^2} + \frac{3M_{\rm P}^2}{2\Omega^4} \left(\frac{d\Omega^2}{d\phi}\right)^2$$

$$V_{\rm E} = \frac{V_{\rm J}}{\Omega^4}$$

Example : Higgs inflation

$$\Omega^2 = 1 + \xi_2 \frac{\phi^2}{M_{\rm P}^2}$$

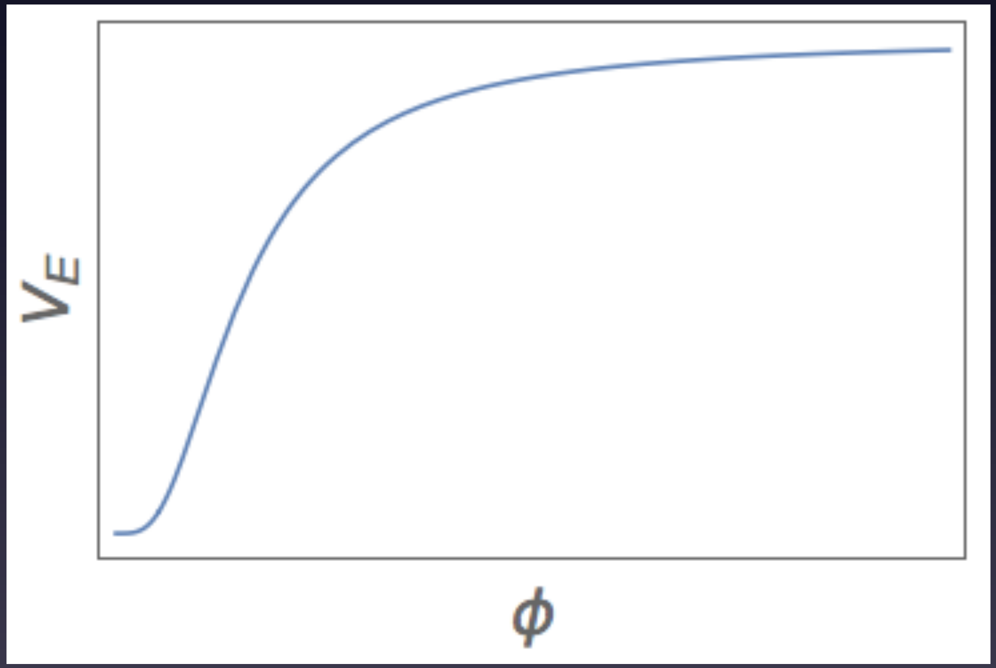
$$Z = 1$$

$$V_{\rm J} = \frac{\lambda}{4} \phi^4$$

Non-minimal Coupling Model

Einstein-frame potential

$$V_{\mathrm{E}} = \frac{\lambda\phi^4}{4(1+\xi_2\phi^2/M_{\mathrm{P}}^2)^2} \longrightarrow \left(\frac{\lambda}{4\xi_2^2}\right) M_{\mathrm{P}}^4$$



Planck normalization (or power spectrum)

$$\frac{\lambda}{\xi_2^2} \sim 10^{-10}$$

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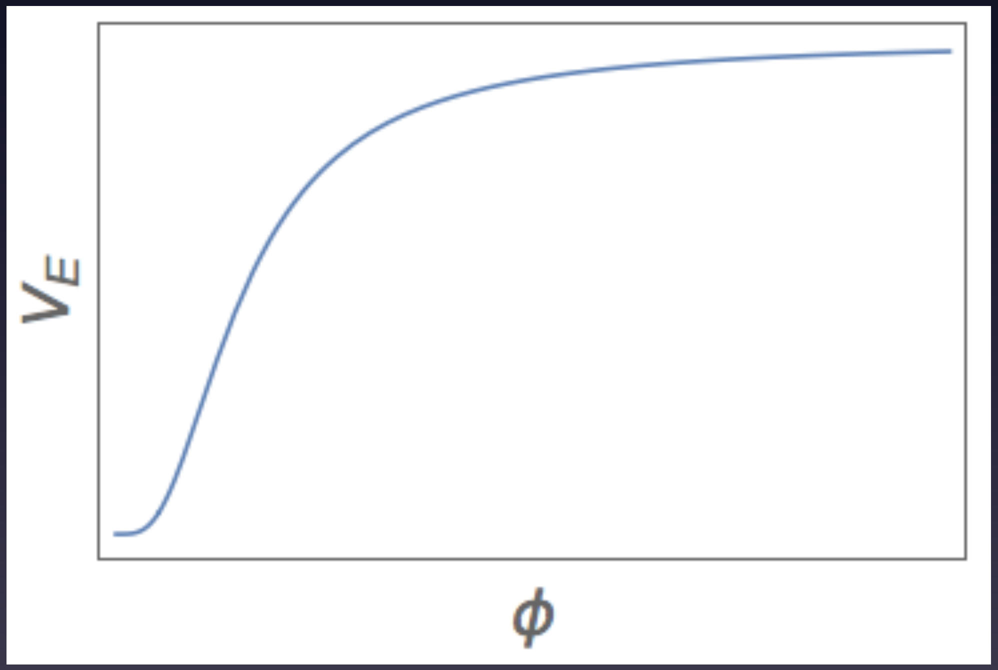

Planck normalization (or power spectrum)

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$$\boxed{V_{\rm E} \ll M_{\rm P}^4}$$?!

Initial Conditions Problem

Require: a potential-dominated initial state over a horizon volume.

Initial conditions for inflation:

the Universe started in a chaotic initial state with Planck-scale energy density

$$\boxed{\frac{1}{2}\dot{\varphi}^2 \sim \frac{1}{2}(\nabla\varphi)^2 \sim V_{\rm E}(\varphi) \sim M_{\rm P}^4}$$

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Plateau inflaton models cannot become potential-dominated during an initial Planck density era.

$$V_{\mathrm{E}} \ll M_{\mathrm{P}}^4$$

Initial Conditions Problem

Several solutions

1. to modify the potential such that it increases as the inflaton field increases and reaches the Planck energy density

2. for a smooth patch to be produced during the chaotic era which has the form of an open Universe, with a negative curvature term which dominates the Friedmann equation

3. to have a contracting era which precedes the expanding era (does not rely on a chaotic initial state)

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Modification of the potential

2 ways :

1. to add non-renormalisable higher-order terms

   → changing the particle physics sector

2. to consider a conformal factor with a zero

   → particle physics sector unchanged

   → regarded as a minimal modification

$$\boxed{V_{\rm E} = \frac{V_{\rm J}}{\Omega^4}}$$

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Conformal Factor with a Zero

Consider a general class of models

$$\xi_2 \frac{\phi^2}{M_{\rm P}^2} \longrightarrow \xi_2 \frac{\phi^2}{M_{\rm P}^2} \times f\left(\frac{\phi^2}{M_{\rm P}^2}\right)$$

Properties of the function f

$$\Omega^2(\phi) \longrightarrow 1 \quad \text{at small} \quad \phi$$

$$\Omega^2(\phi) \longrightarrow 0 \quad \text{at large} \quad \phi$$

$$\boxed{f\left(\frac{\phi^2}{M_{\rm P}^2}\right) = 1 + a_1 \frac{\phi^2}{M_{\rm P}^2} + a_2 \frac{\phi^4}{M_{\rm P}^4} + \cdots}$$

$$a_i \sim \mathcal{O}(1)$$

Conformal Factor with a Zero

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Leading order contributions

$$\Omega^2 = 1 + \xi_2\frac{\phi^2}{M_{\rm P}^2}\times f\left(\frac{\phi^2}{M_{\rm P}^2}\right) = 1 + \xi_2\frac{\phi^2}{M_{\rm P}^2} - \xi_4\frac{\phi^4}{M_{\rm P}^4} + \cdots$$

$$\xi_4 = |a_1|\xi_2 = \mathcal{O}(1)\times\xi_2$$

Remark

The observable predictions of this class of model depend only on the quartic term in the expansion at small field value, i.e., they are independent of the precise form of the function f.

Conformal Factor with a Zero

Conformal factor

$$\Omega^2 = 1 + \xi_2 \frac{\phi^2}{M_{\rm P}^2} - \xi_4 \frac{\phi^4}{M_{\rm P}^4}$$

Einstein-frame potential

$$V_{\rm E} = \frac{\lambda \phi^4}{4(1+\xi_2\phi^2/M_{\rm P}^2 - \xi_4\phi^4/M_{\rm P}^4)^2}$$

Pole

$$\phi_{\rm c} = \frac{M_{\rm P}}{\sqrt{2\xi_4}} \left(\xi_2 + \sqrt{\xi_2^2 + 4\xi_4} \right)^{1/2}$$

Initial condition $V_{\rm E} = M_{\rm P}^4$

$$\phi_{\rm IC} = \frac{M_{\rm P}}{\sqrt{2\xi_4}} \left[\xi_2 - \frac{\sqrt{\lambda}}{2} + \sqrt{\left(\xi_2 - \frac{\sqrt{\lambda}}{2} \right)^2 + 4\xi_4} \right]^{1/2}$$

Conformal Factor with a Zero

Conformal factor

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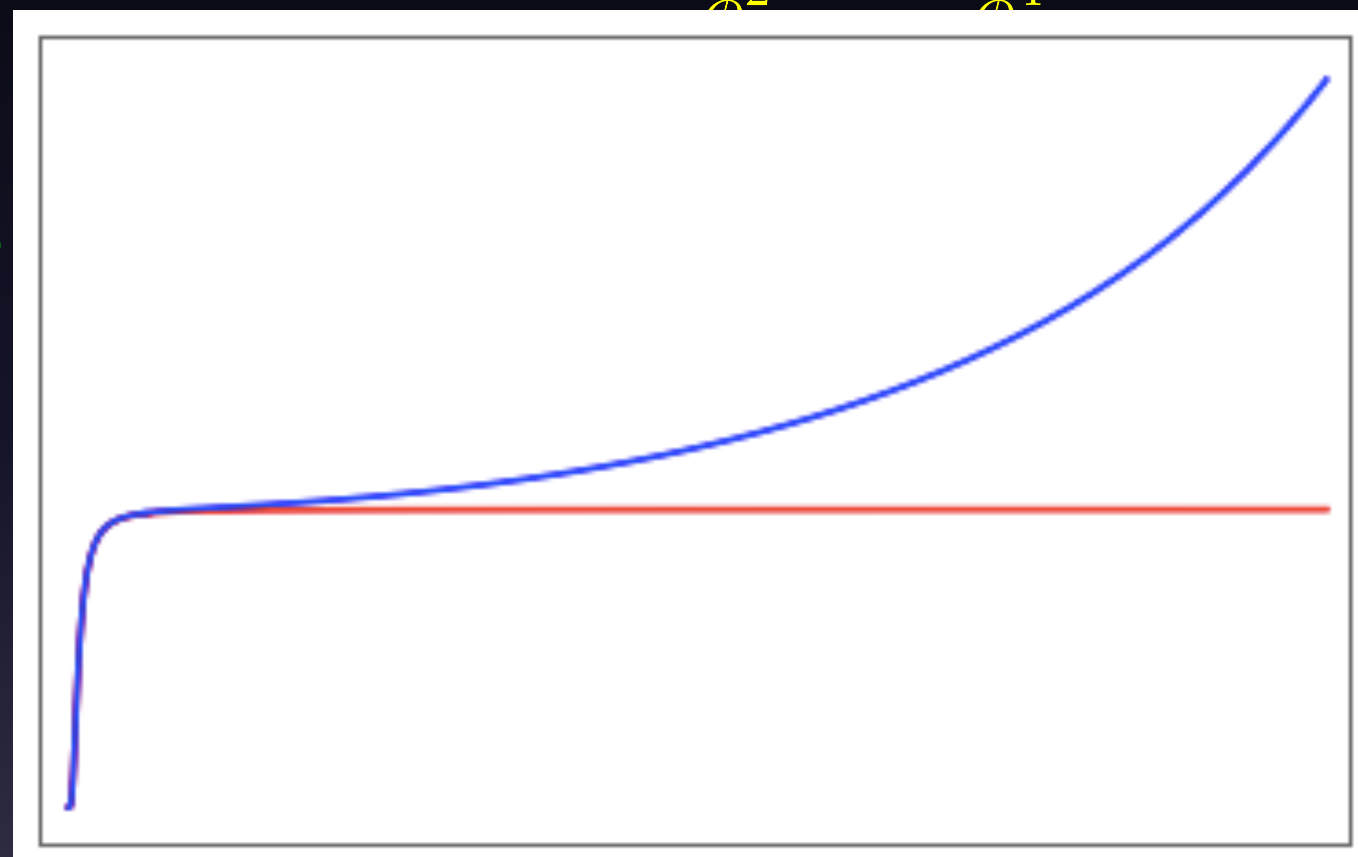

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Conformal Factor with a Zero

At the chaotic initial field value, the potential in the Einstein frame is steep with

$$\rho_{\rm kin} \equiv \frac{1}{2}\dot{\varphi}^2 = M_{\rm P}^4 \qquad \rho_{\rm grad} \equiv \frac{1}{2}(\nabla\varphi)^2 = M_{\rm P}^4 \qquad V_{\rm E} = M_{\rm P}^4$$

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It is therefore important to check :

1. Kinetic energy density ?
2. Gradient energy density ?

Conformal Factor with a Zero

1. Kinetic energy density $\rho = \rho_{\rm kin} + V_{\rm E}$

$$\frac{d\rho}{dt} = \left[\ddot{\varphi} + \frac{dV_{\rm E}}{d\varphi}\right]\dot{\varphi}$$

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{dV_{\rm E}}{d\varphi}$$

$$M_{\rm P}\frac{d\rho_{\rm kin}}{d\varphi} = \sqrt{3\rho}|\dot{\varphi}| - \sqrt{2\epsilon}V_{\rm E}$$

$$\boxed{\rho_{\rm kin\ max} = \sqrt{\epsilon/3}V_{\rm E}}$$

$$\epsilon \equiv \frac{M_{\rm P}^2}{2}\left(\frac{dV_{\rm E}/d\varphi}{V_{\rm E}}\right)^2$$

Conformal Factor with a Zero

2. Gradient energy density

$$\varphi(t,\mathbf{x}) = \overline{\varphi}(t) + \delta\varphi(t,\mathbf{x}) = \overline{\varphi}(t) + \delta\varphi_k(t)e^{i\mathbf{k}.\mathbf{x}}$$

$$\lambda = 2H(0)^{-1} \Longleftrightarrow k = \pi H(0)$$

Conformal Factor with a Zero

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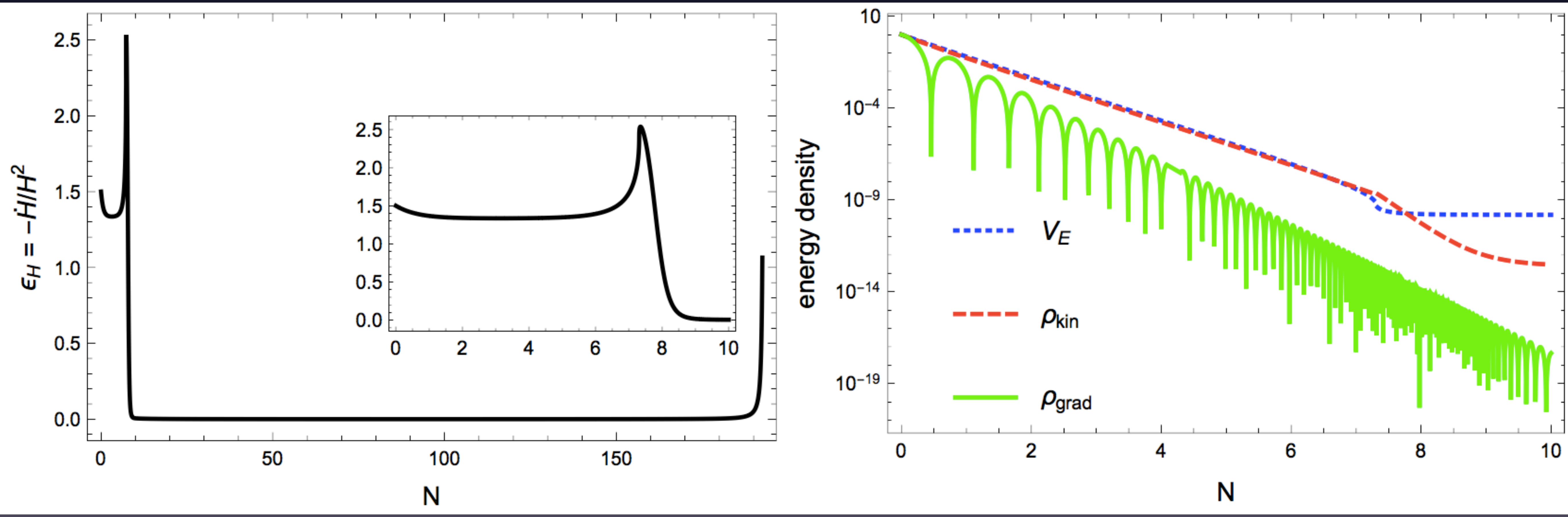


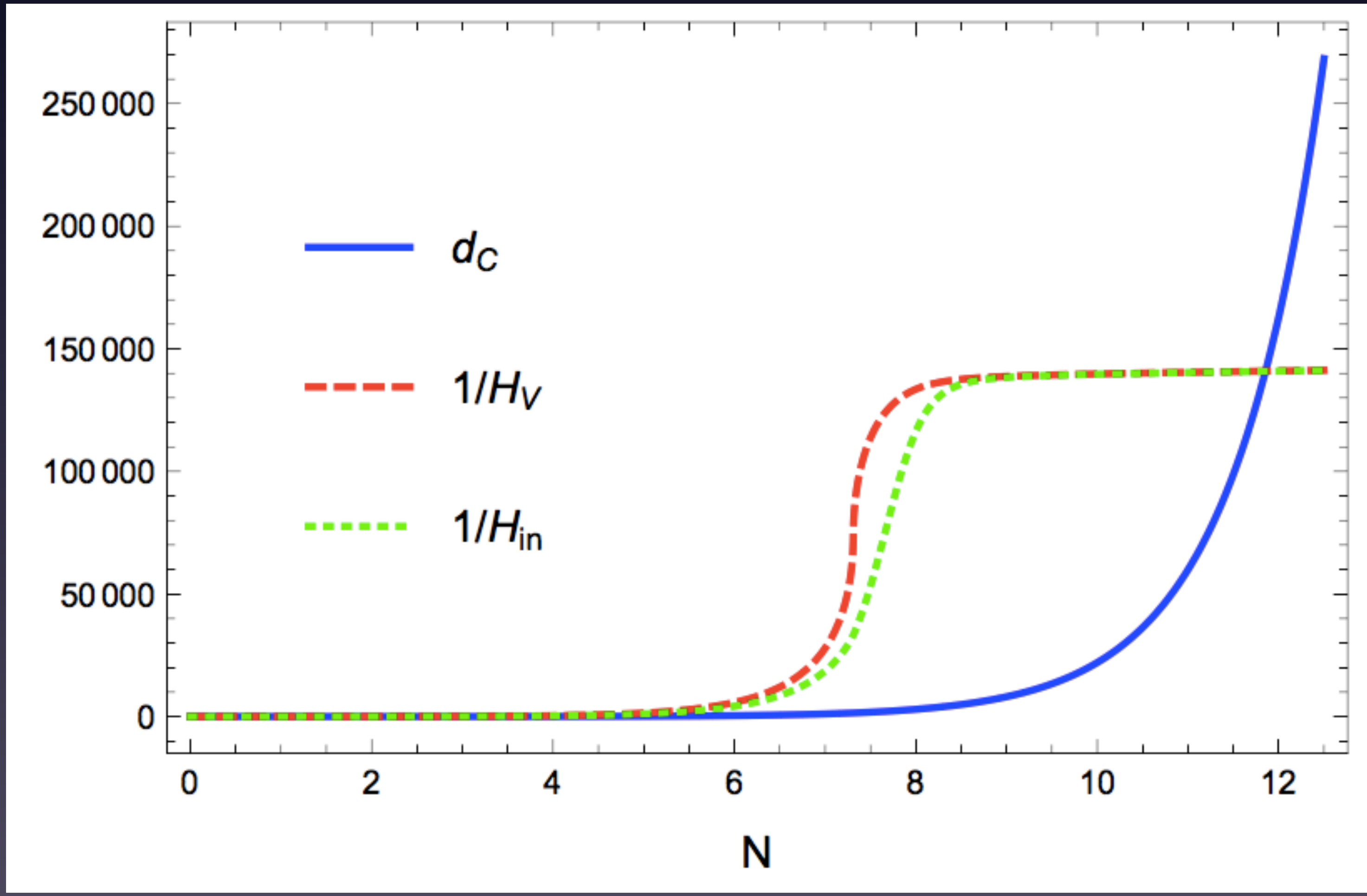
Conformal Factor with a Zero

Horizon

$d_{\mathrm{c}} \equiv a M_{\mathrm{P}}^{-1}$: diameter of the classically evolving volume

$H_{\mathrm{V}}^{-1} \equiv \left(\frac{V_{\mathrm{E}}}{3M_{\mathrm{P}}^2} \right)^{-1/2}$: Hubble radius calculated from the potential energy density

H_{in}^{-1} : Hubble radius calculated using the energy density inside the classically evolving volume



Cosmological Observables

Conformal factor

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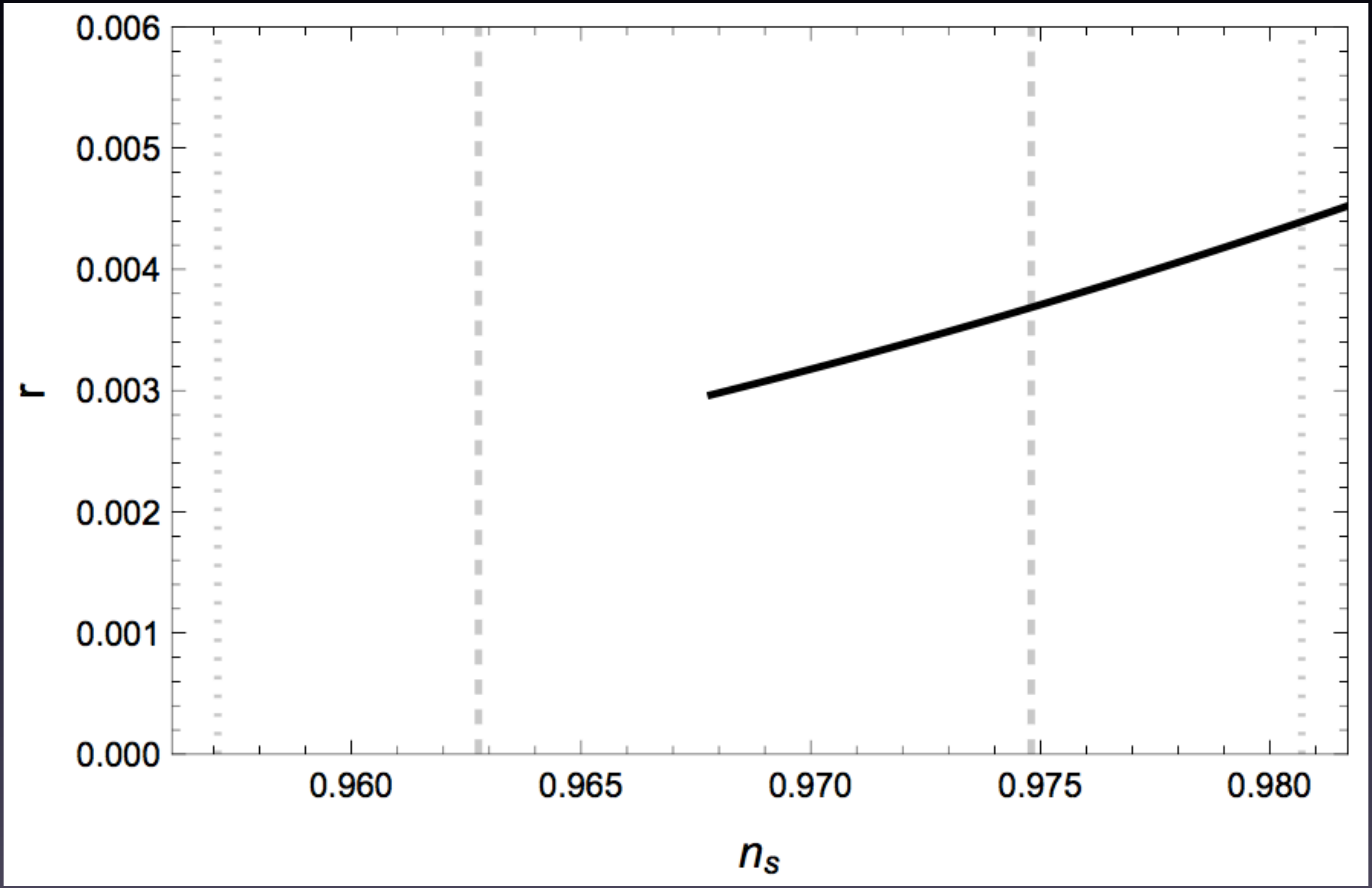
Standard non-minimal model

$$\xi_4 = 0$$

Cosmological Observables



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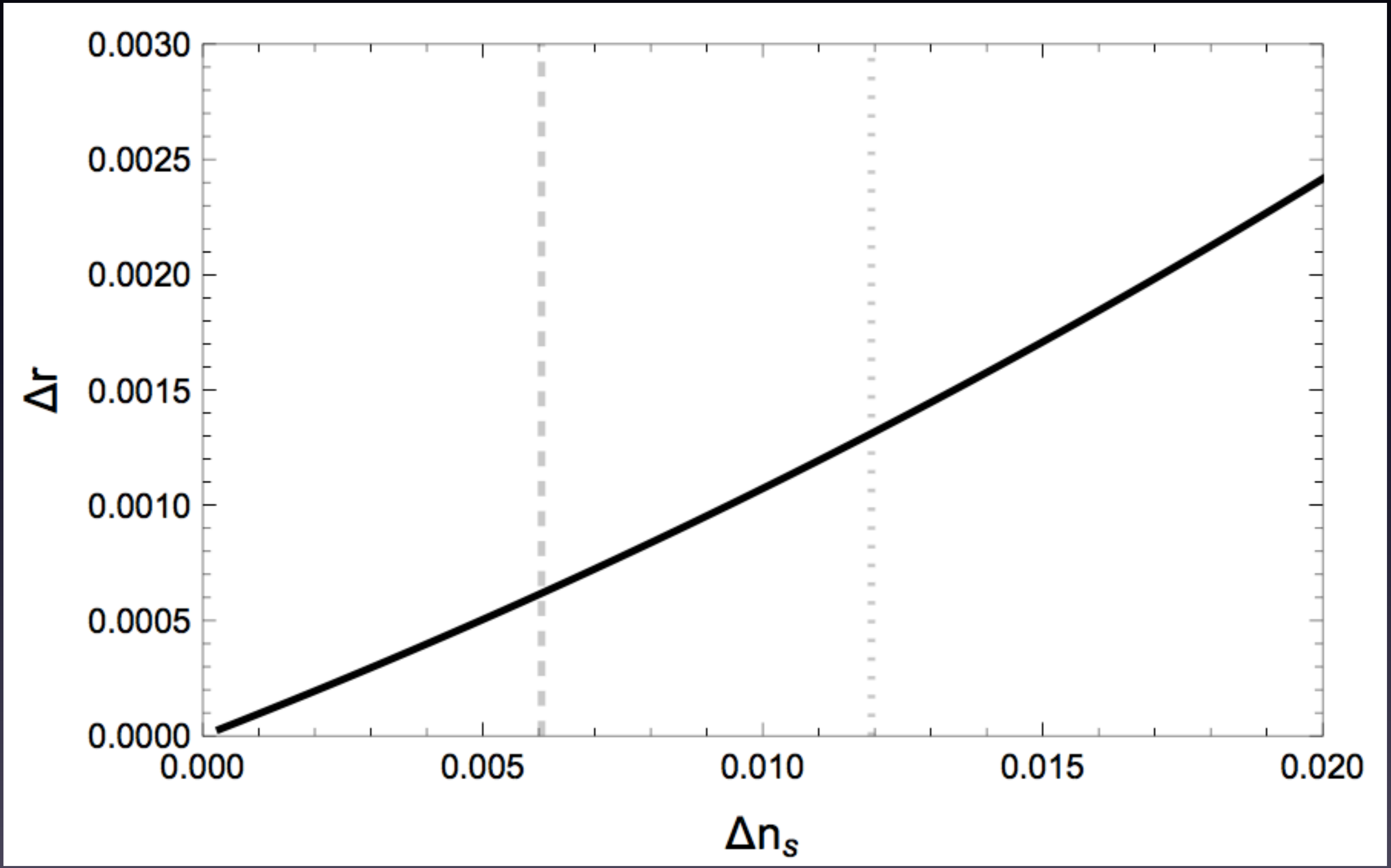

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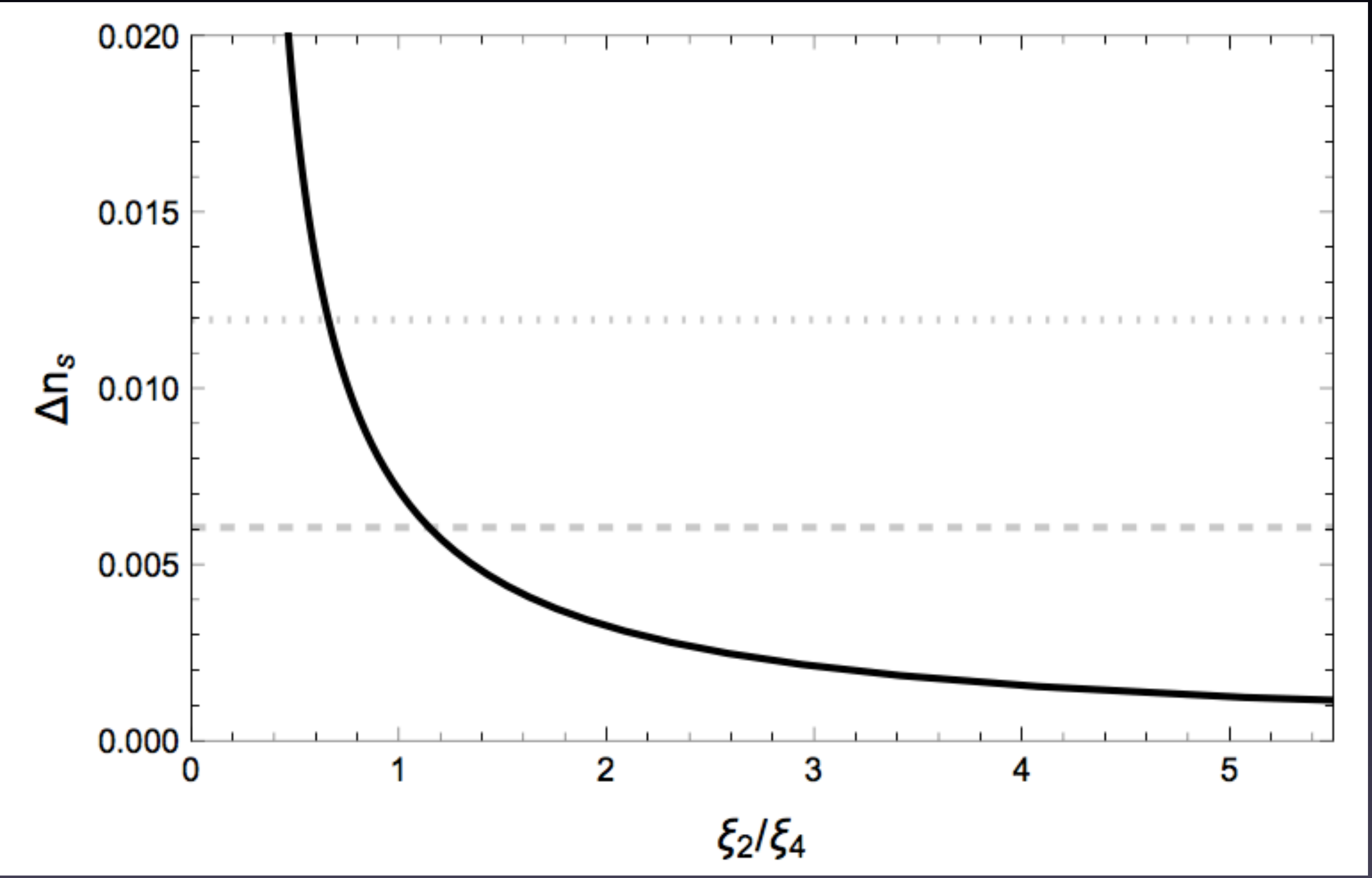

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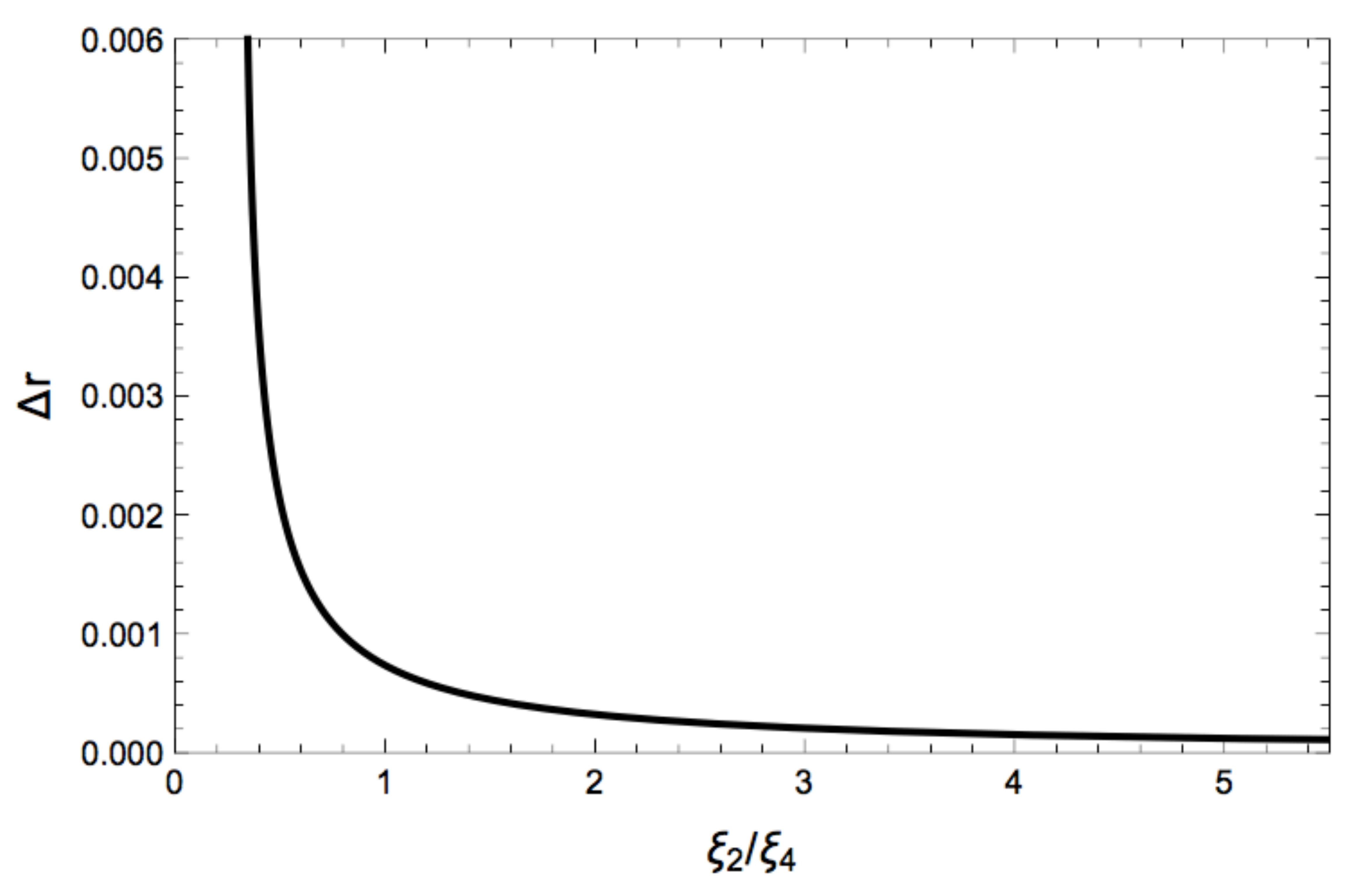

Summary

Non-minimally coupled scalar inflation is in excellent agreement with observation, but it requires an explanation of how inflation got started in the first place.

The chaotic initial state with Planck-scale energy density is not possible for the standard non-minimally coupled inflation model.

By modifying the conformal factor of the standard model to a conformal factor with a zero, it is possible to achieve a Planck potential energy density.

An increase of the tensor-to-scalar ratio as much as 0.0013 is possible.

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Thank you